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# Wave Climate of the North Atlantic - 1970

H. J. A. Neu Report Series/BI-R-76-10/November 1976



# BEDFORD INSTITUTE OF OCEANOGRAPHY

Dartmouth, Nova Scotia Canada

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# WAVE CLIMATE OF THE NORTH ATLANTIC - 1970

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by

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# Atlantic Oceanographic Laboratory Ocean and Aquatic Sciences Department of the Environment

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REPORT SERIES



HMCS Bonaventure, a Canadian aircraft carrier, in heavy sea northeast of the Azores on 6 December 1959. During the height of the storm, winds were 34 m/s (68 knots) with gusts up to 45 m/s (90 knots) and waves reached heights of over 15 m with some exceeding 18 m. Compliments of the Department of Defence, Ottawa.

#### ABSTRACT

Based on synoptic wave charts issued twice daily by the Meteorological and Oceanographic Weather Centre, Halifax, the wave climate of the North Atlantic has been established for the year 1970. The criteria and concepts for developing such a climate are critically reviewed. Representative parameters and statistical terms are used to characterize the sea state.

The results indicate clearly that the sea state, across the North Atlantic, varies greatly with season and location. During the winter, the wave energy was 6 to 10 times greater than during the summer, and in the middle and northeastern Atlantic 4 to 6 times that of the western Atlantic. The largest wave, 21 m in height, occurred near the west coast of Ireland. Based on wave heights, there was 2 to 12 times more wave activity in the northeastern part of the ocean than in the western and southern part of the Atlantic. Long-term probability statistics, based on the 1970 data, indicate that the 'design' wave (100-year wave) varies from 14 to 24 m along the east coast of North America and from 28 to 36 m along the coast of Europe.

#### SOMMAIRE

En se fondant sur les cartes synoptiques des vagues émises deux fois par jour par le Centre de météréologie et d'océanographie d'Halifax, on a établi la description générale des vagues de l'Atlantique nord pour l'année 1970. On étudie de façon critique les critères et les concepts d'élaboration d'une telle description. On utilise les paramètres et les termes statistiques représentatifs pour caractériser l'état de la mer.

Les résultats indiquent clairement que l'état de la mer varie considérablement dans l'Atlantique nord selon la saison et le lieu. Au cours de l'hiver, l'énergie des vagues a été de six (6) à dix (10) fois supérieure à ce qu'elle avait été pendant l'été et, dans l'Atlantique centre et l'Atlantique nord-ouest, elle a été de quatre (4) à six (6) fois supérieure à ce qu'elle a été dans l'Atlantique ouest. La plus grande vague, d'une hauteur de 21 m, s'est produite près de la côte ouest de l'Irlande. Lorsqu'on se fonde sur la hauteur des vagues, on constate que, en matière de vagues, il y a eu de deux (2) à douze (12) fois plus d'activité dans la région nord-ouest de l'océan que dans les régions ouest et sud de l'Atlantique. Les statistiques de probabilité à long terme, fondées sur les données de 1970, indiquent que la vague "type" (vague pour une période de 100 ans) varie de 14 à 24 m le long de la côte est de l'Amérique du Nord et de 28 à 36 m le long de la côte de l'Europe.

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#### 1. INTRODUCTION

The North Atlantic's reputation as a severe and relentless body of water has always been highly respected by mariners. Punishing waves and violent storms have wrecked or sunk countless ships in the past and, despite modern technology, still continue to do so. However, the urgent need for food, minerals and energy impels man to move more and more into the ocean to develop new resources. Ocean exploration, therefore, has been accorded a high priority by many nations.

Wave activity is the greatest single problem in the exploitation of the ocean as a whole and of the North Atlantic in particular. In the oil industry the design criteria for drilling platforms, as well as their towing and placement, are governed entirely by waves. Drilling for oil and its recovery, supply and transfer operations, and the laying of storage tanks and pipes are also at the mercy of waves. The size of drilling platforms has grown to such an extent that some weigh up to 25,000 tons. Down-time of these units is very expensive. It is assumed by the oil industry that there is probably more gas and oil beneath the sea than man has discovered on land. Large reserves, however, are thought to be in depths greater than has yet been reached. Trends in the offshore oil industry indicate that future exploration will be carried out in deeper water and under more severe sea conditions.

The future transportation of oil across the Atlantic will require a large increase in the tanker fleet. With the exception of Canadian seaports, there is no port along the eastern coast of North America capable of handling supertankers. The only solution to this problem is to construct offshore deep-water facilities, but their planning, operation and safety will also be critically affected by waves.

The shortage of energy has accelerated the development of atomic power stations in the coastal region. In the U.S., the floating offshore reactor, which must be protected from waves and ships is favoured at present.

The facts are clear. All present and future activities in the North Atlantic urgently require a more comprehensive knowledge of the sea state. This information is of vital importance to the management of our ocean resources.

#### 2. AREA OF STUDY

The investigation covers the Atlantic Ocean between the 25th and 70th degree north latitudes, that is, roughly between the Tropic of Cancer and the Arctic Circle. As shown on Figure 1, this area includes most of the North Atlantic Ocean but excludes major embayments such as the Gulf of Mexico, Gulf of St. Lawrence, Hudson Bay, Norwegian Sea, North Sea, and Mediterranean Sea. Due to lack of sufficient wave data, the triangular area east of the Madeira and Canary Islands has also been excluded from the study.

With the exception of the areas over the continental shelves, the depth of water in the North Atlantic basin varies from 1000 m to more than 5000 m. Thus, over most of the North Atlantic, waves propagate in 'deep water,' their motion being unaffected by bottom topography. However, on the shelves depths



Figure 1. North Atlantic Ocean

are usually between 200 m and 75 m. The largest of these shelf areas is the Grand Banks of Newfoundland, which extends more than 500 km into the Atlantic. Long waves propagating over this shallow region experience shoaling and refraction, as demonstrated in Figure 2 where refraction diagrams of 14 s waves approaching from the southwest and southeast are shown. The orthogonals in the diagram represent rays lying normal to the wave crests and they indicate the direction of wave propagation and the transfer of wave energy. As can be seen there are areas of energy convergence where the wave height will be increased. This local build-up occurs a few times annually over an integrated period of half to one day and is not directly comparable with the wave conditions of the adjacent deep-water region. Similar modifications occur over Georges Bank at the entrance to the Bay of Fundy, over the Scotian Shelf, west of the U.K., and particularly in the nearshore regions where the decreasing depth toward the shore affects almost the entire ocean wave spectrum.

The North Atlantic stretches over more than 4500 km in any direction. However, storms over the Atlantic rarely extend over distances greater than 1000 km and locally do not exceed 24 hr in duration. Therefore, only a part of the North Atlantic is affected at any one time; and this will limit the growth of the sea state.



Figure 2. Wave refraction over the Grand Banks of Newfoundland

The relationship between the wave dimensions, wind speed and direction, atmospheric stability, fetch, and wind and fetch variability is not very well known, though it has been established that the size of wind-generated waves is determined primarily by the strength of the wind, its duration and fetch. The growth of the sea state with respect to these variables has been investigated by many researchers. On the basis of several hundred measurements in the North Atlantic, Pierson and Moskowitz (1964) derived an equilibrium spectrum which describes, quantitatively, the growth of waves in this area. Figure 3, reproduced from Inoue (1967), shows the development of energy spectra under a uniform wind of 23 m/s, as functions of fetch length and duration of wind. It is demonstrated that for this wind speed, which is typical of an average Atlantic storm, the spectral energy density does not increase appreciably after a fetch length of 1000 km and a wind duration of 2<sup>4</sup> hours have been reached.



Figure 3. Growth of power spectrum with fetch and duration, after Inoue (1967)

As waves leave the storm area where they were generated, their character changes. The crests become lower, more rounded and the lengths longer. Such waves are now called swell. In this form they can travel for thousands of kilometres across the ocean with little loss of energy and can enter any region of the North Atlantic without being related to local storms. Their speed of propagation depends on the wavelength or wave period; the greater their length or the longer their period the greater their speed.

An interesting example was observed by Barber and Ursell (1948). At Cornwall, England, the simultaneous arrival of waves from two different storms was recorded, one system was from a storm which occurred seven days previously south of Newfoundland and the other from a storm near Cape Horn which had occurred nine days previously (Fig. 4). The storm south of Newfoundland generated swells with a period of about 10 s while the more distant storm in the southern ocean produced swells with a characteristic period of about 20 s. The two storms had occurred at distances of about 5000 km and 12,000 km respectively.

Finally, the actual sea state is the result of locally wind-generated waves being superimposed on swell. There are usually a number of swell trains,



Figure 4. Swell of two distant storms at Cornwall, England, after Barber and Ursell (1949)

each with a different direction, present at the same time; as more of these wave fields interact, the more random is the sea surface.

In this investigation no distinction is made between the different wave trains which form the sea state.

#### 3. WIND AND WEATHER

Waves are a kinetic energy system generated by atmospheric winds. These winds result from inequalities in barometric pressure which is basically the result of non-uniform heating of the global atmosphere under solar radiation. The strength of the wind is proportional to the pressure gradient, and its direction is approximately parallel to the contour of the isobars.

For the North Atlantic, the monthly average sea-level pressures for January, April, July, and October are shown in Figure 5 (from the Climatological and Oceanographic Atlas for Mariners (1959). As can be seen, there are distinct differences in the atmospheric pressure patterns during the various seasons of the year. Generally, however, throughout the year higher pressures dominate in the south and lower pressures in the north. High pressure tends to accumulate in a subtropical zone at about latitude 20° to 30°N while low pressure occupies the region between latitude 60° to 65°N. In January, at the height of winter, the low pressure zone in the north is intense and well defined between Greenland and Iceland and is referred to as the Icelandic Low. During the rest of the year it fills in, expands and extends far into the Canadian north. Also in





OCTOBER

JULY

January the high pressure zone in the south, called the Azores High, extends from Florida to Spain, while in summer it intensifies and moves toward the centre of the Atlantic and about  $5^{\circ}$  northward.

The subtropical high pressure belt forms a great wind divide. The light Easterlies or Trade Winds are formed along the south side of the belt. Within the high pressure region are zones of variable winds and calms referred to as the Horse Latitudes. To the north of this region, i.e. between  $35^{\circ}$  and  $60^{\circ}$ N, is the zone of prevailing westerly winds, a most important feature of the North Atlantic wind field. Wind strengths are determined by the pressure difference between the Icelandic Low and the Azores High. As indicated by the spacing and direction of isobars in Figure 5, the wind is much stronger during the winter than during the summer. In January, its direction is from the north-northwest on the western side of the Atlantic, gradually changing over the mid-Atlantic so that on the eastern side it is from the west-southwest. In July the respective directions are from west-southwest and west. North of latitude  $60^{\circ}$  to  $65^{\circ}$ , easterly winds, referred to as the Polar Easterlies, prevail.

The foregoing description of atmospheric motion relates to long-term averages. In middle and particularly in high latitudes, the movement of cyclones and anticyclones, however, brings large day-to-day fluctuations in atmospheric pressure and thus in wind strength and direction. This is especially so during the winter months when a strong flow of very cold Canadian continental air encounters the moist warm air over the Atlantic. The resultant great exchange of energy along the edge of this cold front, referred to as the Polar Front, produces frequent storms. As shown by Sovetova (1969) in Figure 6 for January 1958, almost invariably these disturbances originate along the North American seaboard. The major storm track is parallel to the coast and then heads towards Iceland, where the storms reach their greatest intensity.



Figure 6. Trajectories of North Atlantic cyclones for January, 1957, after Sovetova (1969)

It should be noted that the longest fetch distances occur on the righthand side of the storm paths. Consequently, the greater storm waves of the North Atlantic should be encountered east of the storm route, that is, well off the coast of North America and approaching the west coast of the British Isles. These islands, lying to the right of the normal path of the winter storms, are usually hit with the full force of these wave fields.

The average number of cyclones during the period from October to March exceeds 200. In summer the contrast between conflicting air masses is not as great with the result that the frontal weather disturbances are generally less severe. Depressions are therefore less intense and winds much more gentle.

There is, however, another low pressure system in late summer and early fall. This most notorious type of cyclone is the tropical hurricane which forms in the Trade Wind zone, south of the Azores' high pressure ridge, and follows characteristic tracks as shown on Figure 7. The annual occurrence rate is between 6 and 10. The typical hurricane system has a diameter of about 600 km, approximately half that of a mid-latitude depression, and its full fury is seldom experienced much more than 150 km from the centre. Therefore, in spite of high winds, which can be in excess of 180 km/hr and the resulting local destruction on water and land, the wave-generating capacity for the Atlantic as a whole is not excessive. The exception, which may occur once in a hundred years,



Figure 7. Hurricane tracks for the 1963 season, after U.S. Navy (1964)

is when the propagation speed of the hurricane is similar to that of the wave field it produces. Theoretically, under this condition, the waves would become extremely high.

#### 4. WAVES

A number of research facilities around the world are engaged in wave studies. However, many of the investigations, such as the Joint North Sea Wave Project [JONSWAP] off the island of Sylt, are directed toward basic research into problems of air-sea interaction, the generation of waves, etc. Very little has been done to describe the synoptic sea state across a body of water like the North Atlantic over a period of a month, a year, or more, with the exception of the U.S. Naval Oceanographic Office (1963) and Hogben et al. (1967). Both investigations, however, are based on random observations rather than time series, making a statistical analysis for long-term maximum wave properties impossible. Using hindcast methods, a very comprehensive effort was made in 1962 by the New York University (Bunting, 1966) which developed a computer model for a wave-spectrum climatology based on 12-hourly sea-level pressure data. However, the inability to obtain adequate wind fields across the ocean in order to understand and accurately model the random processes involved has provided results that sometimes have been misleading. A wave climate study, similar to that reported here, has been produced by Neu (1971, 1972) for the Canadian Atlantic coast.

According to the author's interpretation, a wave climate is the description of the average sea state with respect to time and space. The sea, however, is complex and, on occasion, chaotic. Its pattern is never repeated or duplicated. It is evident, therefore, that the problem of describing the sea state of the North Atlantic can be approached at present only by simplistic means.

In this investigation, waves are defined by the following properties:

a. height - H or  $H_V$ , in metres between crest and succeeding trough as seen by an observer

- b. period T or  $T_V$ , in seconds between crest and succeeding crest as seen by an observer
- c. direction  $\alpha$ , in degrees; angle between true north and the mean direction from which waves are coming

#### 4.1 Wave Observation Methods and Data

There are four methods by which wave data are usually obtained: hindcasting from wind observations; direct measurement with wave gauges; aerial survey from aircraft and satellites; and visual observations from ships.

Hindcasting of waves is a rational attempt to solve the problem, but at present it is not very reliable. Its shortcomings lie primarily in the fact that insufficient meteorological observations are made over the open ocean particularly with respect to varying winds and moving fetches.

Direct surveying with wave gauges appears to be the most logical method, yet gauges, exposed to the hazards of the open sea, have only short operational lives. For a time series presentation covering the entire North Atlantic hundreds of such gauges would have to be in operation for years - a monumental task. Unmanned buoys in the Atlantic for sensing waves and telemetering the data via satellite to a shore computer (in real time) would be an intermediate solution but the number of stations would have to be much greater than for a meteorological grid.

Remote sensing, particularly from satellites and HF radar systems, may eventually become practicable and provide instantaneous data for all the oceans of the globe. This is at least 10 to 15 years in the future and, when operational, will be extremely expensive especially in the analysis of the data, as indicated by Pierson (1976).

The only wave data currently available, and which are obtained simultaneously at a number of points across the entire North Atlantic, are those from ships. They are visual estimates of height, period, and direction of waves.

Four times daily, wave and weather observations at 30 to 40 stations consisting of weather ships, Canadian and U.S. Government and Navy ships, merchant ships and oil drilling platforms are radioed to the Maritime Forces Weather Centre in Halifax, N.S. Here the data are reviewed on a 12-hour basis, and related continuously to the preceding and current wave and wind environment by personnel trained in meteorology. Information which does not fit into the developing pattern is checked for errors in observing, reporting, or communicating and, if found faulty, discarded. By this process, wave data are subjected to a fairly strict quality control. The synoptic data are plotted on charts and lines of equal wave height are drawn at one-metre intervals as shown on Figures 8 and 9. In this way temporal and spatial coherence and continuity are imposed across the entire North Atlantic. The charts form a continuous time series of synoptic sea state information reported every 12 hours and these form the basic data for this investigation.

The question of whether two samples per day are sufficient for a wave climate was studied by Thompson *et al.* (1972) on Atlantic City gauge records.



Figure 8. Wave Chart of 26 January 1970



Figure 9. Wave Chart of 27 November 1970

Sampling done once or six times per day provided little variation in the longterm statistics over a year. The data base would, therefore, appear to be adequate.

As mentioned, the wave data used in this investigation are visual esti-According to Wiegel (1964), Ippen (1966), and others, the height given mates. by this type of observation is very close to that defined as the 'significant wave', this height (H ) being the mean height of the highest third of all the waves in a wave train. It is therefore assumed that the visually observed wave height H\_ is equal to H\_. This was verified for the long-term distribution with instrumental recordings by the Marine Environmental Data Service of Environment The sea state was measured with calibrated accelerometer buoys for Canada. 20 minutes every 3 hours, between 5 February and 17 June 1974 at the SEDCO H oil exploration platform near Sable Island, about 75 km off the coast of Nova Scotia. The significant wave heights (H ) were calculated from energy spectra. Utilizing a log-normal distribution plot, in Figure 10, the percentage exceedance (i.e. the percentage of time that a given wave height will be exceeded) of H\_ is compared with that obtained visually (H,) for exactly the same time period. As can be seen, the two lines differ only slightly, the visual height being 7 to 8% smaller than that obtained from wave recorders.



Figure 10. Visual Observations versus Recorded Measurements

In the open ocean the distribution of waves is usually 'random.' Several investigators, in particular Longuet-Higgins (1952), have established that a Rayleigh relationship exists between a known wave height such as the significant height (H<sub>)</sub> and the largest wave (H<sub>max</sub>) for a given number of waves. According to this relationship and as verified by results obtained off the coast of Nova Scotia (Vandall, 1976), the ratio of H<sub>v</sub> to H<sub>max</sub>, for a record in excess of 2 to 6 hours duration, lies between 1:1.7 and 1:2.0. The ratio of 1:1.8 was chosen since it agrees with Thom (1971). The visual wave period T is not directly associated with the visual or significant wave height; it is more the average estimate of all the periods in a wave train. Apparently its magnitude is 15 to 25% lower than the zero-crossing period which is the average period of a wave record. The period more closely related to H is the period at the peak of the wave energy spectrum. This value T is particularly important for the design of coastal engineering structures and the maintenance of inlets and beaches. Correlating the peak periods T taken from the SEDCO H exploration platform in 1974 with the visual values  ${}^{P}T_{v}$  at the same percent of occurrence, the following relationship was obtained:

$$T_{p} = 1.82 T_{v} - 2.41$$
 (1)

For the design of marine structures such as ships, oil exploration and recovery platforms, it is essential to be aware of the largest period which may occur with long term extreme wave heights. Using the equation for  $T_{max}$  (H<sub>s</sub>) developed by Vandall (1976) and the long term relationship between  $T_v$  and H<sub>s</sub> as given by Sonu (1975) the following approximation was established between H<sub>v</sub> (1 year) and  $T_{max}$ :

$$T_{\max} \approx 5.7 \sqrt{H_{v(1 yr)}}$$
(2)

This approximation will be used in this report for determining the long-term extreme periods.

The third, and in many respects the most important, wave parameter is the direction of wave propagation. There are many engineering and scientific problems which cannot be solved without the knowledge of this property. Only those waves which strike a breakwater or a deep-sea terminal are of interest to the designers.

Until now, no practical method existed with which to measure wave directions on the ocean surface over an extended period, except visual observation from ships or platforms. The wave charts of the Maritime Forces Weather Centre (Fig. 8 and 9) contain this information for each ship-reporting station.

## 4.2 <u>Sea State Description Methods</u>

There are two basic types of wave statistics: one is the 'significant wave' type (Sverdrup and Munk, 1947; Bretschneider, 1952; Darbyshire, 1952) and the other is the 'spectral' type (Neumann, 1953; Pierson *et al.*, 1955; Hasselmann, 1973; and others). The first is usually used for long-term time series of representative wave parameters as a characteristic of the sea state for a specified time, e.g., 20 minutes, 6 hours, or even a number of years, while the second is a detailed short-term description of the sea state, usually from a 15- to 20minute long record. In the latter case real-time records are then processed to provide energy spectra which represent the distribution of the wave energy per unit area of sea surface for each frequency range.

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As mentioned, the significant wave concept is more frequently applied by civil engineers in the study of beach processes, design of terminals and breakwaters, in determining long-term extremes, etc., while the spectral method is important to the structural engineer in designing oil drilling platforms, ships, and other ocean structures.

It is obvious that a comprehensive wave climate description should incorporate both methods, one complementing the other. Unfortunately, synoptic instrument wave records do not exist except from a few places along the coast and some weather ships, while visual wave data are quite readily available for the entire North Atlantic and contain, in addition, valuable information on wave direction.

Both types of statistics, however, are compatible, since each, in principle, follows the Rayleigh distribution function suggested by Longuet-Higgins (1952). From this relationship and a predetermined empirical spectrum, the short-term wave spectra can be obtained. There are a number of semi-theoretical spectra to choose from and improvements are being added. Wiegel (1970) has suggested a 'standard' set of spectra such as developed by Scott (1965).

#### 4.3 Long-term Extreme Waves

In both coastal and open ocean operations, annual and longer term extreme wave heights are of utmost importance. Their occurrence is governed by the laws of probability and chance. A technique for obtaining these values has been described by Draper (1963). It is essentially the same as that used in the calculation of extremes of many other terrestrial phenomena, following work by Gumbel (1954) and Jenkinson (1955). This statistical treatment has been applied to the maximum wave heights of each 12-hour period for the entire year of 1970 and plotted on a probability scale against percentage exceedance. The long-term data such as 10-year, 50-year, or 100-year maximum wave heights are then obtained by extending the line of best fit to the required percent exceedance value.

Even though the individual visual observations may not be absolutely accurate, treating the cumulative data statistically provides reliable results. In most instances, the single extreme observation of one year's time series is considered to be the annual maximum wave height. However, if this observation deviates from the straight line through the results in the log-normal graph, it is assumed that this height is either a longer or shorter term value or a bad observation. The intersection of the straight line approximation with the annual occurrence is taken as the correct value.

# 4.4 Wave Energy

To evaluate the relative energy distribution and its seasonal and directional variation across the North Atlantic, the energy of the wave representing the sea state of a 12-hour period was calculated. The energy per wavelength and per unit width of wave crest in the metre-ton-second system is given by:

$$E = 1/8 \gamma H^2 \cdot \lambda [m \text{ ton/m}]$$
(3)

where  $\gamma$  is the specific weight of sea water (approximately 1.025 [ton/m<sup>3</sup>]), and  $\lambda$  is the wavelength (1.56 T<sup>2</sup> [m] in deep water, where T is the wave period in seconds). The energy per metre wave crest per second therefore is:

 $E_0 = 0.2 \cdot H^2 \cdot T [m \text{ ton}]$ 

. . . .

In the analysis, waves were grouped with respect to periods in intervals of 2 s; that is to say, the period T represents the interval from T - 1 (s) to T + 1 (s).

#### 4.5 Grid System

A grid, consisting of ninety-two 5x5-degree 'areas' was placed over the North Atlantic. The total region covered extends approximately from the Tropic of Cancer to above the Arctic Circle with the exception of the region around the Madeira Islands, Canary Islands, and the North Sea. The 'areas' were identified as shown in Figure 11. Also given on this figure are code numbers, their ranges for wave height and period, and the directional system used. It should be noted that the direction convention refers to the direction from which the waves arrive. The same convention is used in meteorology for winds.

For each 'area' and each 12-hour time interval, the mean height and, if available, the period and direction of the observed wave were determined from the published wave charts. The total number of representative waves for each block was 730, which covers as a time series, all of 1970.

Because wave period observations east of 30°W and south of 25°N were less frequent and systematic than the wave height observations, it was decided to restrict the presentation of data, which depend on or include periods such as wave energies and certain statistics, to the western side of the Atlantic. In Figure 11 the area is enclosed with a dashed line. Wave height occurrences and extremes, however, cover the entire North Atlantic.

#### 5. RESULTS

The wave data were grouped in monthly intervals and statistically arranged for computer analysis. The results are presented as follows:

- (1) monthly non-directional energy distribution,
- (2) monthly directional energy spectra,
- (3) monthly directional wave statistics (tabular) of height, periods, and occurrences,
- (4) exceedance diagrams,
- (5) maximum wave height and period distributions for 1970,
- (6) 10- and 100-year (design wave) extreme wave height distribution based on 1970 data,



Figure 11. Grid System and Code for Wave Properties

#### (7) percentage exceedance distribution of given wave heights.

Since the data and results are too many to be reported here in detail, only the distribution of important parameters and representative samples is given. The data on which the analysis are based and the intermediate results of the analysis for each 'area' of the North Atlantic are published in a data report by Walker (1976).

#### 5.1 Monthly Non-directional Energy Distribution

The monthly significant wave energy per metre of wave crest for each 'area' is shown in Figures 12a and 12b. The height of the shaded part of an 'area' is proportional to the energy concentration. From the presentation, it



• • •

JANUARY



FEBRUARY



MARCH



APRIL



Figure 12a. Monthly Nondirectional Energy Distribution, January to June 1970













SEPTEMBER

OCTOBER



Figure 12b. Monthly Nondirectional Energy Distribution, July to December 1970

is seen that the wave energy varies greatly with time and space. As expected, the greatest energies occurred during the winter and the smallest during the summer. In January, the monthly energy level in the northern part of the Atlantic was 5 to 8 times greater than in July. In the southern part, the January/July energy ratio was between 2 and 3 off the U.S. coast while in the centre of the Atlantic it was between 10 and 20. The latter large ratio is due to the low energy level in this part of the ocean during the summer.

Equally as significant as the variation with season is the variation with location. Throughout the year there is an increase in the energy level from west to east as well as from south to north which is particularly significant during the winter months. In January, the energy level rises steadily from the coast of North America toward the mid-Atlantic requiring more than 2000 km in the southern and about 1000 km in the northern part of the Atlantic before the mid-ocean energy level is reached. Eastward from this line the level appears to diminish in the southern part while continuing to increase in the northern part, though at a slower rate. The highest energy level is in the area around 40°N and 40°W, about 1500 km southeast of Newfoundland.

These energy variations with time and space are in good agreement and in phase with the seasonal wind patterns and their distribution. This is shown in Figure 13 by the bimonthly 700 mbar geostrophic wind charts taken from the U.S. Department of Commerce Monthly Weather Reviews (1971). The wind speeds in these charts refer to a height of about 2700 m above the water, which is approximately the upper limit where secondary winds deriving from cyclonic disturbances affect the wind field. The winds given are therefore primary winds, in this case the prevailing westerlies. The correlation between these winds and the wave energy is good. In January, the strongest wind field, marked with letter F, was southeast of Newfoundland where the largest wave energy concentration was also located. In July and in September, stronger wind fields were spread across the ocean from Newfoundland to the U.K. They are clearly detectable in the energy distribution.

Below the 700 mbar height, secondary winds, particularly from cyclonic disturbances, are superimposed on the primary wind field. These winds generate large waves, but for only a relatively short portion of the month (e.g. in January between 10 and 20% of the time), and thus their contribution to the monthly energy level is less dramatic than might be expected.

#### 5.2 Monthly Directional Energy Spectra

The directional energy spectra have been developed for forty-six  $5^{\circ}x5^{\circ}$ 'areas' of the western part of the North Atlantic. A presentation of these data for 12 months would require 552 graphs. To reduce this volume of information only alternate 'areas' were graphically treated and reported in the data report by Walker (1976). Samples are shown here for January only, the stormiest month of the year, for two lines traversing the ocean, one from the coast of Nova Scotia to the mid-Atlantic and the other along longitude  $30^{\circ}W$ , which is approximately the mid-Atlantic (Fig. 14 and 15).

A system based on the eight major directions is used to illustrate the directional distribution of the energy for each 'area.' The integrated significant wave energy is plotted along the directional axis and the wave periods normal to it. The dotted line indicates the percentage of time per month that particular wave periods occur.







MARCH







JULY





Figure 13. Bimonthly 700 mbar Geostrophic Wind Speed (m/s) for 1970 (after U.S. Dept. of Commerce, 1971)



Figure 14. Directional Energy Spectra from the Canadian Coast to the mid-Atlantic in January



Figure 15. Directional Energy Spectra from South to North Along mid-Atlantic in January

As can be seen immediately in Figure 14, the wave energy increases steadily from the coast of North America to the mid-Atlantic. This applies also to the southern part of the ocean. While the wave energy is from west-northwest in the western part of the Atlantic, the direction changes to due west as midocean is approached. This is in agreement with the geostrophic wind deriving from the sea-level pressure shown in Figure 5. The graphs indicate that there is hardly any wave energy from the easterly direction, except from cyclones, the energy of which is too small to show up on the monthly mean values.

Along the mid-Atlantic (Fig. 15), the wave energy first increases from south to north in the southern part of the ocean with most of the energy approaching from the west, but the distribution and direction change in the northern part of the North Atlantic. Here the wave energy is distributed almost equally from the eight directions, then becomes predominantly northeasterly between Iceland and Greenland. This transition into the northeasterly direction is in agreement with the Polar Easterlies which prevail in this region. In 'area' G-10, some wave energy is shown coming from the southwest which is small compared with that from the northeast. This wave energy includes waves from cyclones which, as indicated by the occurrence curve, are few but may embody some of the largest waves in the North Atlantic. July is the month with the lowest wave energy. Its values are extremely small compared with those of January (see 5.1) and are therefore not shown.

## 5.3 Monthly Directional Wave Statistics

The directional wave statistics have been tabulated for the same fortysix  $5^{\circ}x5^{\circ}$  'areas' of the western part of the North Atlantic as have the monthly directional energy spectra. All forty-six tables are reported in the data report by Walker (1976). They contain for each wave representing a 12-hour sea state, the significant height, the visually observed period, number of occurrences, and the predominant direction. The total number of representative waves per month for each area is twice the number of calendar days. The occurrence of a particular combination of height and period is given numerically at the intersection of the two parameters in the statistics tables.

Of the 552 tables for the western North Atlantic, four samples for January 1970 are given on Figure 16. They include one 'area' off the coast of Nova Scotia and three along the mid-Atlantic. Their directional energy spectra have also been shown in Figures 14 and 15.

These tabulations, or scatter diagrams, can be used for further analysis. For instance, the wave steepness, defined as the ratio of wave height to wavelength, may be readily obtained from the tables.

#### 5.4 Exceedance Diagrams

Following the technique used by Draper (1963), the distribution of the significant wave heights and the periods were plotted on log-normal paper to yield the percentage of height and period exceeding any given values. 'Best-fit-lines' were drawn and with practically no exception straight lines were found to fit the data for the 92 'areas' of the North Atlantic.



Figure 16. Directional Wave Statistics for January

An example of such a diagram is shown in Figure 17. The 'area' is C-5 located southeast of Nova Scotia. As can be seen, in this region, waves exceeded the height of 2 m 50% of the time and 7 m 0.5% of the time over the year. The largest wave observed was 8.5 m. The data used for the analysis, however, are those of the fitted line which indicates a height of 8.7 m. Wave periods exceed 6 s 30% of the time and 1% of the time they exceed 11 s.



Figure 17. Exceedance Diagrams for Area C-5

The wave height exceedance diagrams cover the entire North Atlantic whereas the period exceedance data are only available for the western North Atlantic. Samples of the wave height distribution for a number of these 'areas' are shown on Figure 18. The spacing between the distribution lines gives an indication of the rate of change in the sea state across the ocean. For example, in the south along AA, there is little change in wave height and exceedance from the West Indies to the mid-Atlantic. Farther north, in the zone of the prevailing Westerlies, large increases occur in the offshore region of the coast of North America which then decrease in magnitude or even reverse farther eastward. The largest increase is off the coast of Canada.

This growth in the sea state is demonstrated in greater detail in Figures 19a and 19b, where the wave height exceedance diagrams of row E are plotted on a semi-log scale. It is clear that the major increase in wave height occurs between E-6 and E-8 while east of E-8 the growth declines. E-12 obviously is the region with the greatest wave activity while east of it the general sea state declines except for waves in excess of 6 m. 'Area' E-14 at the coast of Ireland shows a marked reduction in the average wave height but an increase in the large wave heights. This indicates that between E-12 and E-14 there is a point where, from the viewpoint of wave generation, the peak of the power spectrum is reached and conditions to the east only broaden its base (see Fig. 3).



Figure 18. Samples of Wave Height Exceedance Diagrams Across the Atlantic



Figure 19a. Wave Height Exceedance Diagrams from West to East



Along the mid-Atlantic (Fig. 19b) wave heights and exceedance increase from south to north reaching a maximum at 'area' D-9. As in the example of Figure 19a, a regrouping in the distribution can be noticed with a decrease in the smaller and increase in the larger waves.

#### 5.5 Maximum Wave Height and Period Distribution

In Figure 17 the distribution line for the maximum wave heights of 'area' C-5 is plotted. As mentioned, this line is obtained by assuming that the maximum wave height of a 12-hour period is 1.8 times larger than the related significant wave. For the largest annual significant wave height of 8.7 m, this yields for 1970 an extreme wave height of 15.7 m.

Based on the results of the 92 'areas,' distribution charts for the largest annual significant and maximum waves of the North Atlantic were developed and are shown in Figures 20 and 21 respectively.

The wave heights given on these charts were generated primarily by the cyclonic disturbances of the winter season. Of the total number of storms, 21% occurred in December, 42% in January, 14% in February, and the remainder during the transition periods in spring and autumn.

There is a well-defined pattern of wave height distribution across the North Atlantic. The smallest waves are south of  $30^{\circ}$ N and the largest, with an H of 21 m, are at the west coast of Ireland. There is a general increase in heights from south to north and from west to east. A region of high waves covers most of the central part of the North Atlantic and extends northeast to the coast of Europe. Superimposed on this region are two zones of exceptionally



Figure 20. One-year (1970) Largest Observed (Significant) Wave Heights

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Figure 21. One-year (1970) Maximum Wave Heights

high waves, one stretching across the ocean south of  $40^{\circ}N$  and the other from an area northeast of Newfoundland to the coast of Ireland. The first has maximum wave heights of about 18 m while in the second the heights increase from 17 to 21 m. Between these two zones there is a region, including the Grand Banks and east of it, where the height of the waves is generally about 2 m less (Fig. 21). The reason for this is unknown. There is no apparent decrease in the strength of the wind over this area to cause such a reduction. It is possible that this may be the result of a combination of factors such as bottom friction over the Grand Banks, earlier breaking of waves due to shoaling, currents, and large-scale mixing. Southeast of the tip of Grand Banks, filaments of the Gulf Stream meet and mix with the Labrador Current and form, according to Mann (1967) and others, the North Atlantic Current. A large eddy (Clarke, 1972), 500 km in diameter, is located in the area. The interaction between the currents and the waves may be a factor in this reduction.

It was assumed previously from meteorological considerations that the largest waves in the North Atlantic occur near Iceland. As shown in Figure 21, the height of the extreme waves in this area is about 4 m less than those off the coast of Ireland.

The distribution of wave periods is as random as that of wave heights. This is verified by the straight exceedance line on Figure 17. A plot of the largest observed values across the western part of the ocean is given on Figure 22. They vary between 13 and 14 s. Although they are the largest observed periods, they do not relate to the highest waves of the respective 'area.' They occur throughout the year and are in most cases the periods of swell with wave heights between 1 and 4 m. The periods of the largest observed (significant) waves are given in Figure 23 as the first number of a group of three. They form a clear pattern consistent with that of the wave height. Along the coast of North America, the periods are in the order of 6 s and increase toward the mid-Atlantic as the fetches become longer. Waves with periods of about 10 s occur in the central ocean southeast of Newfoundland and near Greenland.

These results indicate that extreme periods do not normally coincide with the largest heights. Southeast of Newfoundland, where the waves are obviously smaller than in the surrounding ocean, the periods are among the largest in the Atlantic while northeast of Newfoundland, where the larger waves of the North Atlantic are generated, the periods are in the order of 8 to 9 s. In the latter, the fetch of the prevailing winds is not large enough to generate larger waves. However, this feature may change in the eastern part of the Atlantic for which we do not have period data.

Both the observed or significant height as well as the observed period are used as the representative parameters of a 12-hour sea state but, for each 12-hour time interval, there are a variety of periods for any given height and a variety of heights for any given period. Using Eq. (1), the peak period  $T_p$ , that is the period with the greatest concentration of energy in the wave energy spectrum, was determined and using Eq. (2) the probable long-term maximum period that may occur with an observed one-year maximum wave height was obtained. The respective values in seconds are given as second and third numbers in Figure 23. The peak period is an essential property in designing breakwaters, harbours, and studying beach processes, while the maximum long-term period is needed for the design of sea structures such as oil recovery platforms.



Figure 23. Observed  $(T_v)$ , estimated peak  $(T_p)$  and probable long-term extreme  $(T_{max})$  wave periods of highest waves in 1970

# 5.6 10- and 100-year Extreme Wave Heights based on 1970 Data

In building coastal structures and oil rigs, a design life of 10 years or even longer must be considered. For oil rigs, for example, the 'lifetime' wave is usually the 100-year wave. To obtain these long-term values, probability statistics are applied. As shown in Figure 17, this is done by extending the exceedance line for maximum wave heights to the percentage of exceedance for the required time interval. From the results, the 10- and 100-year extreme wave height distributions are obtained which are shown in Figures 24 and 25 respectively.

The patterns, naturally, are similar to that of the one-year distribution, though the wave heights differ appreciably. The 10- and 100-year extremes vary along the coast of North America from 12 to 18 m and from 14 to 26 m respectively, with the lower range being found both along the coast of Florida, and in the northern waters of Canada in Davis Strait and Baffin Bay, and the higher range at the southeastern tip of Newfoundland. As in the case of the one-year data, the largest long-term waves of the North Atlantic are off Ireland with heights of 28 m and 36 m respectively.

The latter estimates were verified by Draper (1972), who analyzed records obtained from wave gauges along the coast of U.K. during the last quarter of a century. According to his results the 50-year most probable maximum wave height along the western coast of Ireland is 33 m.

Extreme value statistics have not been applied to the periods of these waves.

# 5.7 Percentage Exceedance Distribution of Given Wave Heights

For any type of operation in the ocean it is important to forecast the interruptions which can be expected from waves. In oil exploration, the motion of platforms becomes critical when waves are larger than 3 m and drilling is usually stopped when they exceed 10 m. At a height of 20 m platforms are evacuated. To estimate these interruptions, percentage exceedance distribution are developed for the significant wave heights of 1.5, 3.5, 5.5, and 7.5 m and their respective maximum values of 2.7, 6.3, 9.9, and 13.5 m.

As shown in Figure 26, the results clearly indicate that the area with the lowest occurrence is along the coast of North America and across the southern part of the ocean while the area with the largest activity is west of Ireland. For the smaller ranges, the 1.5 and 3.5 m significant wave group, the occurrence at Ireland is four to five times greater while for the larger waves, with significant heights in excess of 7.5 m, it is 12 times greater than on the opposite side of the ocean and in the south.

#### 6. CONCLUSION

Waves are important in almost any phase of ocean activity and this knowledge is becoming crucial to the future management of the ocean's resources. For this purpose a wave climate was developed which describes the sea state of the North Atlantic in a rational and understandable way. Representative



Figure 24. 10-year Maximum Wave Heights



Figure 25. 100-year Maximum Wave Heights (Design Waves )

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Figure 26. Percentage exceedance of Given Wave Heights

parameters and statistical terms were used which characterize the conditions. A most significant feature of the investigation is the directional statistics. This property is all important for engineering applications and scientific studies. These parameters were derived utilizing visual observations from ships which were analyzed with the application of random-process statistics.

Since the sea state arises directly or indirectly from the wind, a more detailed analysis of the seasonal and regional wind field is given. Both the primary winds generated by the pressure difference between the Icelandic Low and the Azores High as well as the secondary winds deriving from cyclonic disturbances along the Atlantic Polar Front show good correlation with the generation and distribution of waves, the first with the monthly patterns and the second with the annual extremes.

From the results, it is obvious that the sea state across the North Atlantic is highly non-uniform temporally and spatially. It is far more severe during the winter than during the summer and waves are much lower in the southern and western part of the ocean than in the northeastern North Atlantic. The

smallest occurrence and the lowest annual maximum waves were encountered along the coast of North America and across the ocean from Florida to North Africa. From these areas and to the east and north respectively, the heights, periods, and occurrences grew rapidly until a high level of activity was reached in the central part of the North Atlantic. From here on the sea state increased only in the northeasterly direction. The highest wave activity was about 1000 km west of Ireland and the largest annual waves of the ocean occurred off the east coast of Ireland with  $H_s = 11.5$  m and  $H_m = 21$  m. The frequency of occurrence of waves is, on the European side, 2 to 12 times greater (depending on wave height) than on the American side.

The distribution of the periods of the highest annual waves shows similar trends for their respective heights. The annual waves are relatively short along the coast of North America but are nearly double in the mid-Atlantic. In contrast to the wave height distribution, the periods are large in the southern part of the North Atlantic.

Extreme value statistics was applied to obtain long-term maximum wave heights. Based on the 1970 data, the 'design wave' (100-year wave) is 14 m for the coast of Florida, 20 m for the region off New York, 22 to 24 m for the coast of Nova Scotia and the Grand Banks, and 36 m for the west coast of Ireland.

Power spectra, which are required for motion-control studies of drilling rigs and ships, can be obtained from this report by applying self-consistent models that provide average one-dimensional sea spectra from long-term height and period statistics.

The wave climate analysis will be continued for 1971 and 1972. Preliminary results indicate that a similarity exists between the data reported herein and those of the following two years. From the combined data, a three-year wave climate will be developed and an effort will be made to provide directional probability statistics for each month of the year.

Wave data from gauge records taken near Sable Island agreed with the long-term visual results within 7 to 8% (Fig. 10). There is also complete agreement with Draper (1972) on the 50-year maximum wave height for the west coast of England and Ireland and with Bores (1974) on the 100-year maximum wave for the coast of Spain which is about 30 m. Thom's 100-year wave heights for the North Atlantic Weather Ships are compared with ours in the table below:

Weather Ship	А	В	C	D	Έ	I	J	K
Thom (1971)	33	29	26	26	24	42	42	34.5
BIO	29	29	30	28	28	32	34	32

There is ageement to within 15% for most of the stations except I and J which appear to be high. For a body of water the size of the North Atlantic, this agreement is outstanding and proves the applicability of the methods.

Finally, in ocean activities, there is still a large gap between reliable predictions from analysis and the needs of the scientists and engineers engaged in the interpretation of the physical behavior of the ocean and in the design of marine structures. An effort is made with this investigation to narrow this gap. One fact becomes increasingly clear from this study: 30-m or 100-foot waves are not just a sailor's yarn - they do exist in the North Atlantic.

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